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*Ultraviolet Spectroscopy of*  
*Planetary Atmospheres*

*Charles A. Barth*

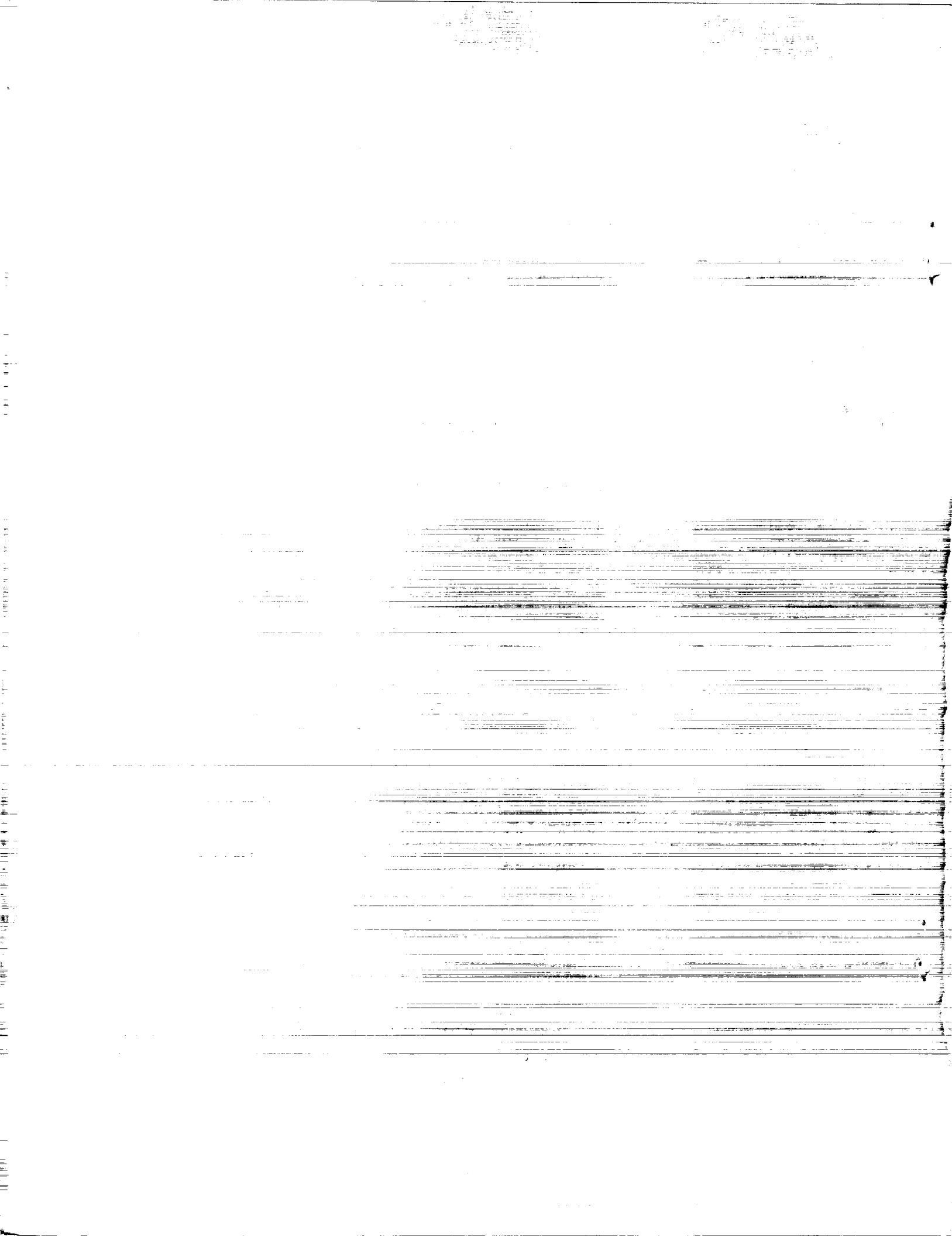
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PASADENA, CALIFORNIA

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# ULTRAVIOLET SPECTROSCOPY OF PLANETARY ATMOSPHERES\*

by

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## ABSTRACT

The ultraviolet spectrum of a planetary atmosphere is produced by charged-particle bombardment and solar radiation. The ultraviolet aurora and dayglow may be observed from a rocket within the atmosphere, from a satellite above the atmosphere, and from a space probe flying by the planet. The spectrum of the ultraviolet dayglow is the result of molecular scattering, absorption, resonance re-radiation, and fluorescence of the incident solar radiation. The composition of the upper atmosphere may be determined from a quantitative analysis of the dayglow spectra. The spectrum of the ultraviolet aurora identifies many of the atoms and molecules that are present in the atmosphere. The geographic distribution of the aurora is believed to be the result of the interaction of the solar plasma with the planet's magnetosphere. The details of the spectrum will provide information on the energy and nature of the bombarding particles.

## INTRODUCTION

Planetary atmospheres exhibit ultraviolet spectra that are characteristic of the atoms and molecules that make up the atmosphere and of the physical processes that excite these atoms and molecules. Ultraviolet planetary spectra may be obtained from rockets flying through the atmosphere or from satellites or space probes flying above the planet. The study of the ultraviolet spectral region provides a unique opportunity to detect the atoms and diatomic molecules that are in the atmosphere.

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Ultraviolet emission spectra have two main sources. The first is the dayglow, which is the result of the solar ultraviolet radiation falling on the atmosphere. The other is the aurora, the result of charged particle bombardment of atmospheric constituents.

#### DAYGLOW

The fundamental physical parameter that determines the spectrum of the dayglow is the spectral distribution of the solar radiation that falls on the atmosphere. In the visible portion of the spectrum, the sun behaves as a black body of about  $5800^{\circ}\text{K}$ . On the short wavelength side of the black body curve, in the spectral region below 3000 Å, the solar radiation curve has been measured in Naval Research Laboratory rocket experiments. The solar intensity distribution between 1000 and 3000 Å which is shown in Figures 1 and 2 is taken from their work<sup>(1)</sup>. Between 1700 and 3000 Å, the solar spectrum is a smooth continuum. Below 1500 Å, solar emission features make up the spectrum. The most prominent of these is Lyman alpha emission at 1216 Å. Another spectral feature of special interest in this wavelength range arises from atomic oxygen at 1300 Å. It is the solar radiation between approximately 1000 Å and 3000 Å that gives rise to the important features in the ultraviolet dayglow.

To use the earth's atmosphere as an example, the impinging solar radiation is absorbed by ozone in the spectral range between 2000 and 3000 Å and by molecular oxygen in the spectral region between 1300 and 2000 Å. Figure 3 shows the absorption coefficient of ozone versus wavelength which has been measured in the laboratory by Inn and Tanaka<sup>(2)</sup>. The maximum absorption occurs at approximately 2500 Å. Figure 4, which is from the work of Watanabe and his collaborators<sup>(3)</sup>, shows the absorption coefficient of molecular oxygen versus wavelength in the wavelength interval between 1000 and 1750 Å. Above 1750 Å, oxygen absorbs discretely into the Schumann-Runge bands as may be seen in Figure 5. Below 1750 Å, the Schumann-Runge continuum reaches a maximum near 1400 Å. Below 1300 Å, there are various windows that appear in the absorption spectrum. Because of absorption by molecular oxygen and ozone, solar radiation at different wavelengths between 1000 and 3000 penetrates to different levels of the earth's atmosphere. Figure 6 shows the height to which the solar radiation of various wavelengths is able to penetrate in the earth's atmosphere. More precisely, the curve indicates the level at which the intensity of incident radiation is reduced to  $1/e$  of its initial value. Between 2000 Å and 3000 Å, solar radiation is able to penetrate down to the

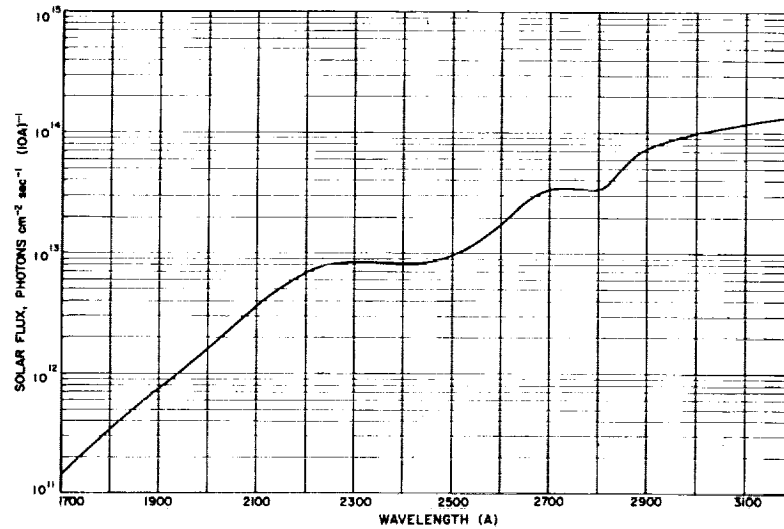


Figure 1. The Solar Intensity Distribution from 1700 – 3000 Å, from Detwiler, Garrett, Purcell, and Tousey (1)

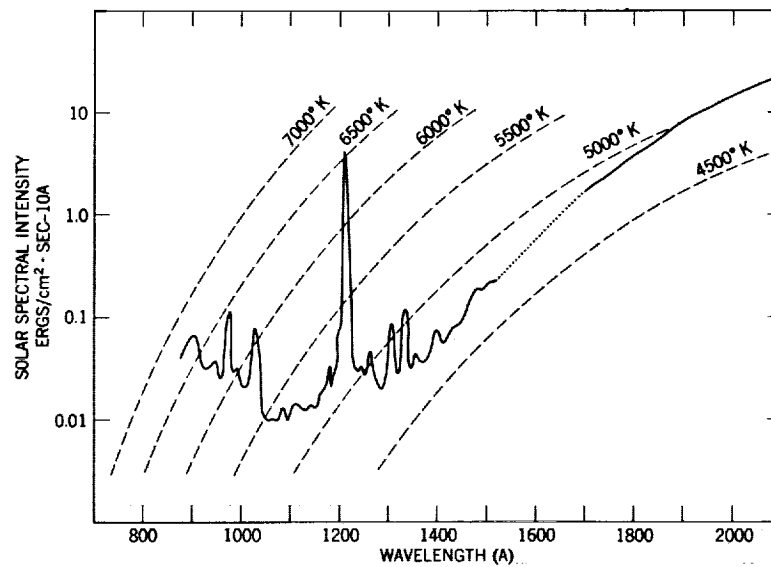


Figure 2. The Solar Intensity Distribution from 900 – 2000 Å, from Detwiler, Garrett, Purcell, and Tousey (1). (Reproduced through the courtesy of Geophysique and United States Naval Research Laboratory)

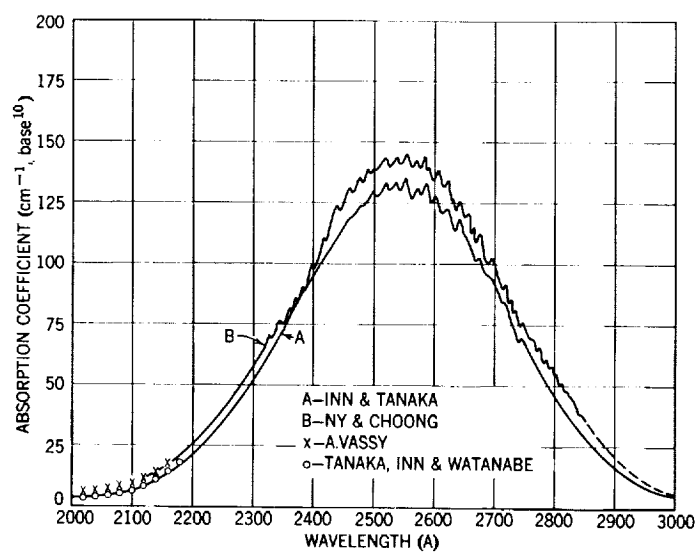


Figure 3. Absorption Coefficient of Ozone in the Region 2000 - 3000 Å, from Inn and Tanaka (2). (Reproduced through the courtesy of the Journal of the Optical Society of America)

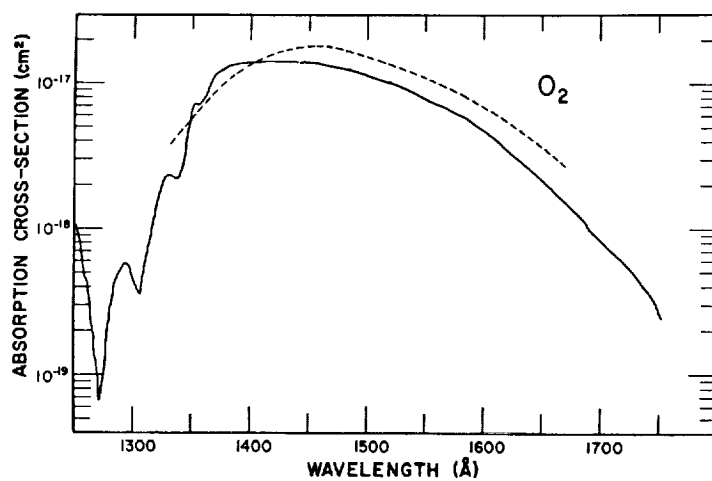


Figure 4. Absorption Cross-Section of Molecular Oxygen in the Region 1250 - 1750 Å, from Watanabe (3). (Reproduced through the courtesy of Academic Press)

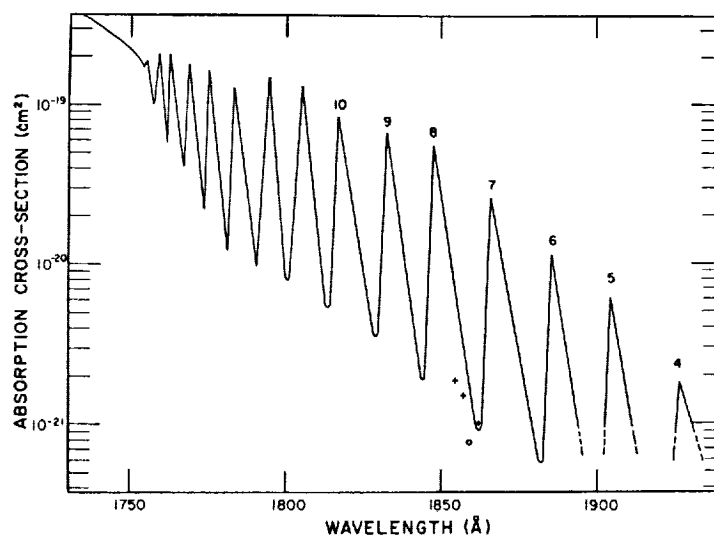


Figure 5. Absorption Cross-Section of Molecular Oxygen in the Region 1750 - 1950 Å, from Watanabe (3). (Reproduced through the courtesy of Academic Press)

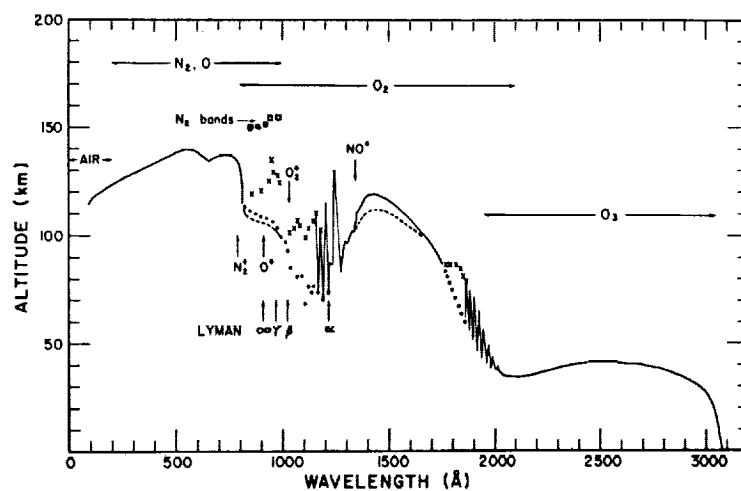


Figure 6. Penetration of Solar Ultraviolet Radiation into the Atmosphere, from Watanabe (3). (Reproduced through the courtesy of Academic Press)



30-40 kilometer region in the atmosphere. Between 2000 and 1750 Å, the discrete Schumann-Runge bands absorb in the altitude range between 30-100 kilometers. Between 1750 Å and 1000 Å, the solar radiation is absorbed above approximately the 100 kilometer level.

The incident solar radiation will be modified by several different physical phenomena when it falls upon the atmosphere. The atoms that are present in the atmosphere will absorb specific wavelengths and then re-emit this energy in resonance re-radiation. The molecules that are present, especially the diatomic molecules, will absorb certain wavelengths of the incident light and then re-emit the radiation in fluorescence. In the lower atmosphere, all of the molecules present will scatter the incident solar radiation through Rayleigh scattering. The effects of Rayleigh scattering will be modified, however, by any absorption that occurs in the lower atmosphere, particularly by polyatomic molecules that may be present.

It is the study of the manner in which these four physical phenomena occur in the atmosphere that makes it possible, from the examination of the ultraviolet spectra, to determine both the composition and the structure of a planetary atmosphere. The usefulness of observing the ultraviolet spectra of planetary atmospheres has been advocated by Chamberlain<sup>(4, 5)</sup>.

In the earth's atmosphere, the principal atomic lines that occur in the ultraviolet dayglow are the Lyman alpha 1216 Å line of atomic hydrogen and the 1300 Å resonance line of atomic oxygen, 3p - 3s. The resonance line is a triplet transition producing three lines at 1302, 1304, and 1306 Å. In other planetary atmospheres such as Mars, resonance lines of other atoms may be present; for example, atomic nitrogen at 1200 Å or atomic argon at 1048 Å. It is characteristic of atmospheric structure that the lighter atoms occur in the outermost part of the atmosphere. This means that when a planetary atmosphere is viewed from afar, these atomic resonance emissions may appear as a corona or halo about the planet. In the case of the earth, the Lyman alpha radiation of atomic hydrogen extends all the way out to one earth radii. The 1300 Å line of atomic oxygen extends approximately 1000 kilometers above the surface. Thus, with a combination of spectral resolution and spatial resolution that is available from a rocket or spacecraft, one can map out the distribution of atoms in the upper atmosphere of a planet.

The diatomic molecules that are present in an atmosphere absorb the radiation at altitudes lower than the atoms. For example, molecular oxygen absorbs between 30 and 100 kilometers. Figure 7 shows an energy level diagram of molecular oxygen. The band system of interest in ultraviolet fluorescence

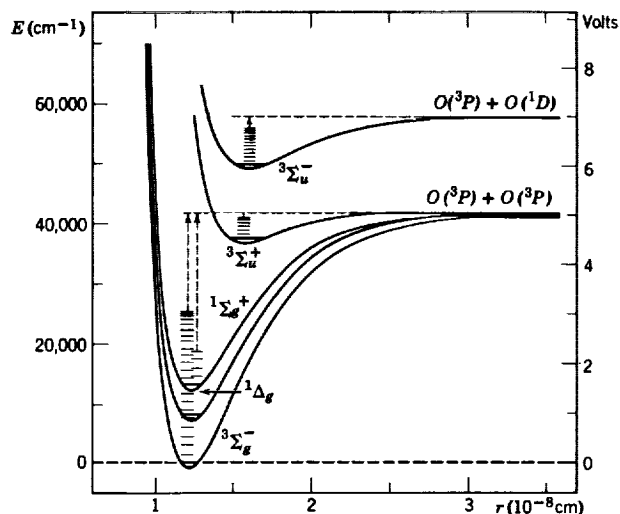


Figure 7. Potential Energy Curves of Molecular Oxygen, from G. Herzberg, *Spectra of Diatomic Molecules*, Van Nostrand Co., New York (1950). (Reproduced through the courtesy of the Van Nostrand Company)

is the Schumann-Runge system,  ${}^3\Sigma_u^- - {}^3\Sigma_g^-$ . Absorption into this system occurs between 1750 and 2000 Å. Molecules that are in the lowest vibrational level of the lowest electronic state populating all of the vibrational levels in accordance with the Franck-Condon principle. Since the upper electronic state has a larger inter-nuclear distance than the ground electronic state, most of the re-emitted radiation will occur at longer wavelengths. Table I shows the Franck-Condon factor array from the work of Nicholls<sup>(6)</sup>, illustrating the situation in molecular oxygen. Absorption occurs into the first column where  $v'' = 0$ . Since the transition probability decreases in going from higher to lower  $v'$ 's, the solar radiation penetrates progressively deeper into the atmosphere for bands originating from decreasing vibrational levels. In fluorescence, each upper vibrational level re-emits all the bands along a row with an intensity proportional to the Franck-Condon factor. The dots across this array show the positions of the "Condon parabolas". The short wavelength energy that is absorbed into bands listed in the first column is degraded to longer wavelengths appearing in the bands listed in the table along the parabolas. The expected fluorescent spectra have been calculated<sup>(7)</sup> and

Table I

Franck-Condon Factors For The Schumann-Range System of O<sub>2</sub> (Reproduced through the courtesy of the Canadian Journal of Physics)

	0	1	2	3	4	5	6	7	8	9	10
0	3.3026-6	9.7126-8	1.3805-6	1.2033-5	7.7180-5	3.7873-4	1.4800-3	4.7221-3	1.2524-2	2.8070-2	5.1261-2
1	3.9075-8	1.0281-6	1.9015-5	1.0219-4	5.7182-4	2.4074-3	7.8835-3	2.0183-2	4.2622-2	7.7841-2	9.8776-2
2	2.4464-7	8.7887-6	6.4721-5	4.5190-4	2.3003-3	8.8550-3	3.1377-2	4.4190-2	6.8887-2	7.8728-2	8.8043-2
3	1.0427-6	2.3437-5	2.2478-4	1.3850-3	5.8435-3	1.7073-2	3.8804-2	4.1591-2	6.7163-2	4.2345-2	6.0216-2
4	3.1798-6	8.7805-5	6.0843-4	3.1011-3	1.0021-2	3.6180-2	5.5090-2	6.1178-2	3.8768-2	6.5028-2	4.8018-2
5	9.5880-6	1.7110-4	1.3745-3	5.5583-3	2.0739-2	1.2735-2	3.4841-2	4.2878-2	1.0217-2	2.7654-2	3.0646-2
6	7.7374-5	3.7335-4	3.6835-3	1.1212-2	2.9558-2	4.0234-2	1.7648-2	1.8713-2	2.1519-2	2.7652-2	2.7144-2
7	4.9711-4	7.9211-4	4.6821-3	1.6081-2	1.1580-2	1.8007-2	3.0840-2	3.8848-2	6.4376-2	7.8007-2	2.0936-2
8	9.0632-5	1.2824-3	7.3428-3	2.7220-2	4.5387-2	4.1577-2	1.3801-2	7.4734-2	1.3072-2	2.8205-2	3.1437-2
9	1.7312-4	3.0880-3	1.0552-2	2.8007-2	4.6419-2	2.9470-2	2.8781-2	5.7863-2	1.3908-2	1.3124-2	1.4096-2
10	2.8030-4	3.1716-3	1.4294-2	3.2722-2	4.0000-2	1.7010-2	9.5813-2	1.0053-2	2.1120-2	7.7140-2	1.1030-2
11	5.5811-4	4.5387-3	1.8281-2	3.6446-2	3.3274-2	6.8646-2	1.1199-2	4.2802-2	1.3329-2	9.7508-2	1.2881-2
12	4.7918-4	6.1581-3	2.1993-2	3.6747-2	2.4773-2	1.1080-2	1.1158-2	2.3748-2	2.7310-2	8.0423-2	2.1135-2
13	9.0760-4	7.9324-3	2.5200-2	3.2844-2	1.6136-2	1.0558-2	1.7611-2	1.7300-2	1.1755-2	5.8820-2	3.3387-2
14	1.7229-3	9.9130-3	2.7864-2	1.1738-2	8.8001-2	2.5852-2	6.0814-2	9.5099-2	2.2594-2	1.7392-2	4.5256-2
15	1.7134-3	1.1923-2	2.9190-2	2.8095-2	3.5517-2	6.9014-2	2.0397-2	3.7778-2	7.6300-2	1.6480-2	2.0545-2
16	2.2239-3	1.3851-2	2.7716-2	2.1815-2	7.0197-2	1.1208-2	1.6638-2	2.3993-2	1.7745-2	1.0823-2	2.7745-2
17	7.5338-3	1.5409-2	2.7129-2	1.0222-2	1.8544-2	1.4841-2	1.1880-2	5.3276-2	1.6564-2	4.0352-2	5.6273-2
18	3.1999-3	1.7190-2	2.8067-2	1.1208-2	1.0010-2	1.8407-2	7.0143-2	5.0086-2	1.4912-2	8.0867-2	1.0193-2
19	3.9072-3	1.8475-2	2.6180-2	7.1790-2	3.0097-2	1.8407-2	3.1183-2	6.4993-2	1.2698-2	6.0498-2	1.9948-2
20	4.4975-3	1.8430-2	1.7882-2	4.0128-2	5.4377-2	1.1157-2	7.7150-2	9.9018-2	8.2158-2	1.6173-2	7.2532-2
21	5.0753-3	2.0044-2	2.0849-2	1.5330-2	7.7488-2	1.7743-2	2.2566-2	1.1848-2	4.4052-2	4.8887-2	9.9817-2

	11	12	13	14	15	16	17	18	19	20	21
0	8.8771-8	1.1151-7	1.6828-7	1.8884-7	1.3788-6	1.0738-6	7.7104-6	4.1588-5	2.0812-5	8.5067-5	3.0411-5
1	8.1021-7	6.2656-6	2.1864-6	1.0787-6	1.0234-6	4.7087-6	1.1971-5	4.3009-5	1.4884-5	7.0084-5	4.1375-5
2	2.0000-6	7.3633-7	2.1708-6	6.7739-6	8.8484-6	5.5661-5	1.9543-5	1.3335-5	7.0036-5	1.1884-5	1.8484-5
3	8.8442-7	3.8436-6	1.6161-6	1.1874-6	1.1282-6	1.7764-6	8.8976-6	2.7744-5	1.7704-5	1.9012-5	6.0640-5
4	5.4097-7	4.8218-6	1.3187-6	1.5192-6	2.3274-6	4.2727-6	1.6873-5	1.1309-5	4.7764-5	4.4757-5	2.8007-5
5	1.0844-6	1.1480-6	2.1180-6	2.4467-6	2.8790-6	7.2800-6	1.3603-5	4.7418-5	2.8363-5	1.2308-5	3.2300-5
6	1.4153-6	1.5408-6	7.9194-6	3.0201-6	1.8808-6	1.6780-6	4.0082-5	1.1688-5	5.0458-5	4.8709-5	1.1771-5
7	1.2605-7	2.0892-6	2.0040-6	2.0113-6	1.3632-6	2.4773-6	8.0048-6	8.8070-6	8.8918-6	1.0710-5	4.9036-5
8	1.0379-7	2.7744-6	1.8661-6	4.2088-6	2.0318-6	0.1138-5	5.7265-5	8.8007-6	8.8918-6	6.0141-5	3.7784-5
9	4.8179-8	1.6808-7	5.8001-6	2.3815-6	6.0070-6	1.7827-6	8.8918-6	1.3365-5	3.0646-5	3.1902-5	9.4771-5
10	2.2880-7	1.8094-6	1.2715-6	2.8488-6	4.5489-6	1.7852-6	1.9016-6	4.1577-5	8.8446-6	1.2418-5	3.9701-5
11	1.2070-7	1.9041-6	6.4871-6	2.2078-6	6.4555-6	8.5036-6	1.6700-6	1.5844-6	1.8289-6	7.0941-6	8.8187-6
12	1.8040-8	1.1752-7	1.7579-7	1.0821-6	1.8977-6	8.7814-6	4.5861-6	8.8909-6	1.1590-6	1.7110-6	1.4938-6
13	2.2904-8	1.8788-8	6.1783-7	6.3863-7	8.5338-7	1.5779-6	1.8908-6	9.3470-6	5.3332-6	8.7218-6	1.1128-6
14	8.8052-8	1.4848-8	6.8505-7	1.3282-7	2.7796-7	4.6217-6	1.7811-6	7.8788-6	1.7448-6	6.9714-6	8.5787-6
15	1.5883-8	6.0774-8	5.8275-7	1.8789-7	5.8279-7	7.9072-7	8.1825-6	1.6218-6	1.3173-6	1.8888-6	
16	1.8971-8	2.1520-8	0.5405-7	0.8063-7	1.1071-7	1.6880-7	1.1515-6	1.2008-6	5.5090-6	7.7287-6	1.2030-6
17	3.3008-8	2.8693-8	1.3698-7	3.3286-7	7.9908-7	1.0168-6	2.0840-6	1.4768-6	1.4220-6	1.4040-6	3.5631-6
18	7.0633-8	1.5308-7	1.3777-7	3.8770-7	8.7780-7	0.0445-6	8.3632-6	8.2711-6	3.6780-6	1.2778-6	6.8417-6
19	2.9867-8	6.9176-8	6.9773-7	1.6432-7	2.4977-7	9.4048-7	1.8175-6	1.0683-6	1.0080-6	5.0872-6	8.8778-6
20	3.1174-8	1.0222-7	2.9232-7	5.7175-7	8.3934-7	1.8941-6	1.1365-6	1.4180-6	1.8990-6	2.9001-6	7.8514-6
21	1.8823-8	1.1274-8	6.9827-8	8.0164-8	2.3367-7	8.8908-8	8.8108-8	9.1888-8	8.4385-8	1.5108-7	1.1182-7

are shown in Figure 8. The solar flux in the appropriate wavelength interval has been multiplied by the albedo which has been derived from the Franck-Condon factors. Each vertical line in this figure should be thought of as a molecular band about 10 Å wide. It can be seen that the spectrum is quite complex and does result in a degradation of the short wavelength solar radiation to longer wavelengths filling the entire spectral interval between 1750 and 3000 Å. Other molecules that may fluoresce in other atmospheres may be the molecular nitrogen Lyman-Birge-Hopfield bands between 1100 and 2600 Å in the atmosphere of Mars, and in the atmosphere of Venus, the fourth positive bands of carbon monoxide between approximately 1100 and 2600 Å. The theory of fluorescent scattering in planetary atmospheres has been described by Chamberlain and Sobouti (8, 9).

In the earth's atmosphere, the solar radiation between 2000 and 3000 Å does not penetrate into the lower atmosphere because of the ozone absorption. However, the molecules that remain above the ozone region do produce Rayleigh

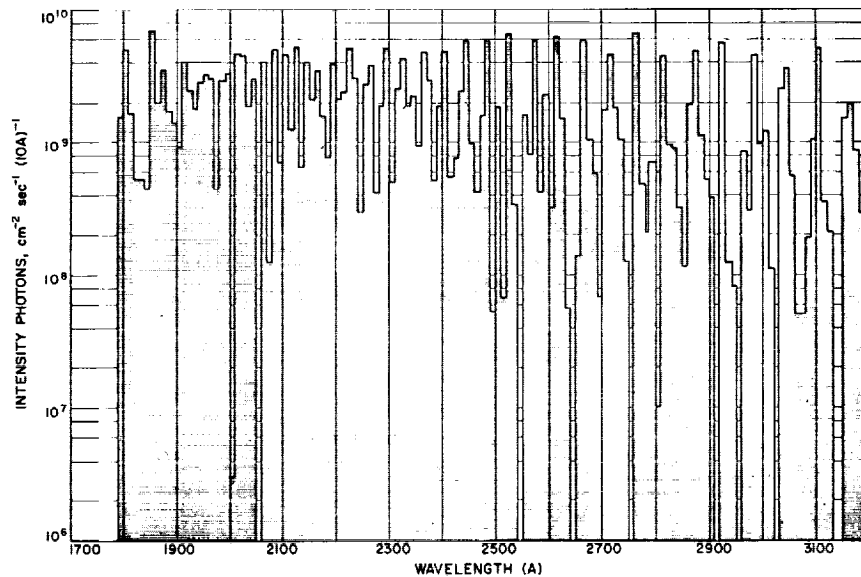


Figure 8. Ultraviolet Dayglow Intensity Distribution Resulting from Fluorescent Scattering, (from Barth and Tobmatsu 87)

scattering and thus contribute to the dayglow spectrum. The molecular scattering coefficient follows a  $1/\lambda^4$  law. Figure 9 shows the calculation of the Rayleigh scattering component of the dayglow looking downward from above.<sup>(7)</sup> The incident flux, the uppermost curve, is scattered by the atmosphere below it. Above 3000 Å, the entire atmosphere above the surface scatters the sunlight, producing a high albedo. The shape of the curve below 3000 Å is the result of the presence of ozone in the earth's atmosphere. The shape of such a curve for an unknown planetary atmosphere is determined by the amount of ozone in the atmosphere.

## AURORA

The aurora is the result of charged-particle bombardment of the upper atmosphere of the planet. A study of the visible spectrum of the earth's aurora shows that electron bombardment accounts for the most pronounced of the spectral features.<sup>(5)</sup> The principal molecular emitter in an aurora is molecular nitrogen. The prominent features that are observable from the ground are the first negative bands of nitrogen, the first and second positive bands, and the Vegard-Kaplan bands. Figure 10 shows an energy level diagram of molecular nitrogen.

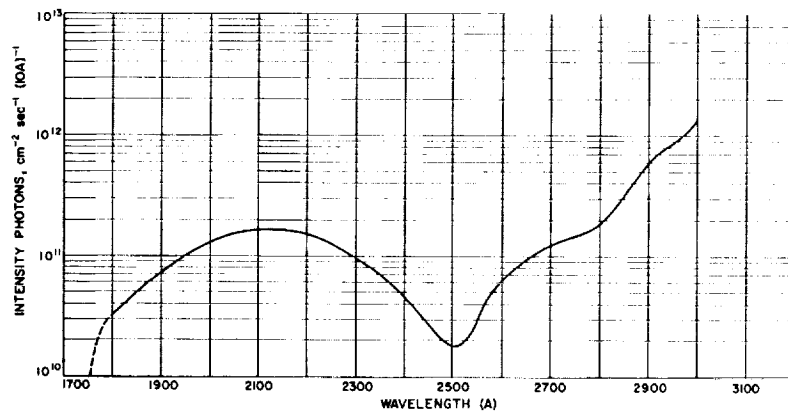


Figure 9. Ultraviolet Dayglow Intensity Distribution Resulting from Rayleigh Scattering, from Barth and Tobmatsu (7)

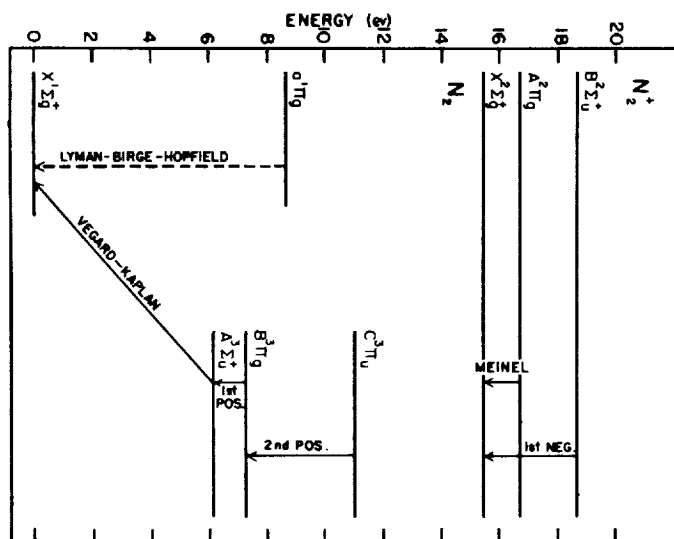


Figure 10. Electronic States and Band Systems of Molecular Nitrogen, from Chamberlain (5). (Reproduced through the courtesy of Academic Press)

The band systems that are seen in the visible spectrum occur from triplet to triplet transitions and are mainly the result of cascading from higher excitations. The Lyman-Birge-Hopfield bands which are excited from the ground state in a singlet-to-singlet transition are not observable from the ground as they occur in the ultraviolet below 3000 Å. Chamberlain<sup>(10)</sup> has pointed out that these bands should carry most of the energy of the aurora. Since they are a singlet to singlet transition, their spectra should give a good indication of the energy distribution of the exciting electrons. The atom emissions in an aurora that have the most intense and the most significant spectra occur in the ultraviolet. For instance, the resonance line of atomic nitrogen  $4S - 4P$  is at 1200 Å.

Proton bombardment in the aurora produces the Lyman alpha line of atomic hydrogen at 1216 Å. The intensity of this line is a measure of the proton excitation process.

Both theoretical and laboratory evidence indicates that the bulk of the auroral luminosity lies in the ultraviolet part of the spectrum<sup>(5)</sup> and that the intensity there will be greater than the intensity of the dayglow. This means that the results of charged-particle bombardment may be observed on both the illuminated and dark portions of a planet by looking in the ultraviolet below 1750 Å. The spatial distribution of the ultraviolet aurora may be obtained from a spacecraft moving by a planet. This spatial distribution will be determined by the interaction of the solar wind with the magnetic field of the particular planet. The observation of auroral emissions from the planet's upper atmosphere thus may be used to infer the existence of a planetary magnetic field.

Table II presents a summary of some of the spectral emissions from dayglow and aurora which may occur in various planetary atmospheres.

Table II  
Atoms and Molecules That May Be Detected In Planetary Atmospheres  
By Ultraviolet Spectroscopy

		Earth	Venus	Mars
Dayglow	Resonance Reradiation	H - 1216 A O - 1300 A	H - 1216 A O - 1300 A	N - 1200 A O - 1300 A H - 1216 A A - 1048 A
	Fluorescence	O <sub>2</sub> - Schumann-Runge 1800 - 3000 A	CO - 4th Positive 1100 - 2600 A	N <sub>2</sub> - Lyman-Birge-Hopfield 1100 - 2600 A
	Absorption	O <sub>3</sub> - 2000 - 3000 A	O <sub>3</sub> - 2000 - 3000 A	O <sub>3</sub> - 2000 - 3000 A
Aurora		N <sub>2</sub> - Lyman-Birge-Hopfield 1100 - 2600 A	N <sub>2</sub> - L-B-H	N <sub>2</sub> - L-B-H
		N - 1200 A	N - 1200 A	N - 1200 A
		O - 1300 A	O - 1300 A	O - 1300 A
		H - 1216 A	H - 1216 A	H - 1216 A

## REFERENCES

1. Detwiler, C. R., Garrett, D. L., Purcell, J. D. and Tousey, R.: *Ann de Geophys.*, 17, 9 (1961).
2. Inn, E. C. Y. and Tanaka, Y.: *J. Opt. Soc. Am.*, 43, 870 (1953).
3. Watanabe, K.: *Adv. in Geophys.*, 5, 154 (1958).
4. Chamberlain, J. W.: *The atmospheres of Mars and Venus*, NAS-NRC publication 944, 147 (1961).
5. Chamberlain, J. W.: *Physics of the Aurora and Airglow*, Academic Press, New York, 1961.
6. Nicholls, R. W.: *Can. J. Phys.*, 38, 1705 (1960).
7. Barth, C. A. and Tohmatsu, T. (1963).
8. Chamberlain, J. W. and Sobouti, Y.: *Ap. J.*, 135, 925 (1962).
9. Sobouti, Y.: *Ap. J.*, 135, 938 (1962).
10. Chamberlain, J. W.: *Memories Soc. R. Sc. Liege*, cinquieme serie, tome IV, 606 (1961).

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